Lesson 8: Making Inferences

Selected Content Standards

Benchmarks Addressed:
D-5-M Comparing experimental probability results with theoretical probability (e.g., representing probabilities of concrete situations as common fractions, investigating single-event and multiple-event probability, using sample spaces, geometric figures, tables, and/or graphs)

D-7-H Making inferences from data that are organized in charts, tables, and graphs (e.g., pictograph; bar, line, or circle graph; stem-and-leaf plot or scatter plot)

GLEs Addressed:

Grade 8
44. Use experimental data presented in tables and graphs to make outcome predictions of independent events (D-5-M)

Grade 10
22. Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, scatter plot, matrix, or circle graph (D-7-H)

Lesson Focus:
In many ways this lesson is a continuation of Lesson Five. Students are asked to analyze graphs within the context of a problem-solving situation.

GEE 21 Connection
Students will be required to:

• Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, or circle graph in context
• Make predictions based on the calculation of the probability of independent event, in a described situation or using experimental data presented in tables or graphs

Translating Content Standards into Instruction

Begin with Teacher Blackline #1. As a class look at (a). Encourage students to tell stories that would match the graph. Point out to them that (a) is a line that starts at (0,0) Ask them about the rate of growth they see. We want them to see that this is a steady rate of growth. Ask what real life situations would generate a graph like (a). Divide the students into groups and ask them to discuss (b). Point out to them that the graph starts somewhere on the y-axis. Ask them to decide what rate of growth they see and to come up with a real-life situation that this graph might model. Follow with each of the other four graphs, asking about the rate of growth and a real world model. If students have trouble with this exercise, present the following situations and ask them...
to match the statement with the correct graph. (The ordered pairs following the statement show the labels for the axes as ordered pairs (horizontal axis, vertical axis), and the answer for the correct graph.)

1. At first the population of nutria grew at an increasing rate. Then the hurricane hit and the food supply became limited. The nutria population continued to grow, but at a decreasing rate. (time, population) (o)

2. Soybean prices are up, so Mr. Johnson decided to plant his 100 acres in soybeans. He estimated that it would cost him $500.00 up front to ready the acreage and purchase some one time supplies. It would then cost him $20.00 per acre for the soybeans and fertilizer needed. (number of acres, cost) (b)

3. Driving along the entrance to the interstate, I accelerated the car in order to enter the traffic at 55 miles per hour. (time, distance) (e)

4. Jeanne fell as she sprang from the starting block, got to her feet and began running the 200 meters. (time, distance) (d)

5. The further the planet is from the sun, the longer it takes to orbit around the sun. (distance, orbit time) (g)

6. The car I drive gets 28 miles per gallon in the city. (gallon, miles) (a)

The students are now ready to discuss the distance/time graphs found on teacher Teacher Blackline #2. The graphs on this master and all other graphs in this lesson were taken from the Book of Graphs put out by the Shell Centre for Mathematical Education, University of Nottingham, 1985. This master shows 9 possible distance/time graphs. Students are asked

- Which graphs can never represent the journey of a single vehicle? Why?
- For the rest make up situations which could cause these graphs to be drawn.

Discuss question #1 as a class. Students need to be able to see that time cannot go back. Therefore, d and h cannot possibly be distance/time graphs. In the case of these graphs distance is interpreted to mean that you can return to where you started. (Answer (b) would be such a graph.) You cannot go a distance in an instant (unless you are Harry Potter) so (i) cannot be a distance/time graph. The teacher might want at this point to break the class into groups to discuss the remaining six graphs and then report to the rest of the class the situations they think that describe the graph they see. Once this set of graphs has been discussed, hand out Student Worksheet #1. In question #1, students are to pick the most likely graph. In question #2, students are to pick the most accurate description.

The last discussion of graphs looks at two graphs together. This graph is a distance/time graph that looks at five seconds of a trip of a car and a motorbike. This graph is found on teacher Teacher Blackline #3. Students should discuss

- The trip of the car; its speed and acceleration
- The trip of the motorbike; its speed and acceleration
- Comparison of the two trips
Some ideas to be brought out include the following:

- The speed of the car remains constant since the graph is a line.
- Students might want to make a table of the distance vs. time and see that the distance is changing at the constant rate of 10 meters/second. There is no acceleration.
- The speed of the motorbike is increasing at an increasing rate during the 5-second period. Making a table of time vs. distance would show this also. At \( t = 0 \) the car is 10 meters ahead and already moving. At \( t = 0 \) the motorbike is not moving. This is shown looking at the \( y \)-intercept of each. The motorbike starts moving and accelerates, increasing its speed with each passing second. The car stays ahead in the first four seconds, but at the fourth second the motorbike catches up with the car and pulls ahead. At \( t = 4 \) seconds, the two vehicles have gone the same distance. In a distance/time graph such as this, the fact that one graph is above the other means that at that point the vehicle has gone farther.

Hand out **Student Worksheet #2: Growth Curves**. Let the students work in groups to answer the questions. Follow up with a class discussion. The final **Student Worksheet #3** is called *Going to School*. Make sure that the students understand that the graph on page 2 is speed/distance rather than distance/time. An answer sheet for this worksheet is included for the teacher.

**Sources of Evidence of Student Learning**

Is the student involved and contributing? You might need to have students write a description of one of the graphs involved to make sure that each sees the necessary ideas. For instance, in the 9-graph distance/time master students should be able to point out that horizontal lines mean that the trip has stopped while lines with different slopes mean that the speed has changed. They should see that the steeper the slope of the line, the faster the speed. In **Student Worksheet #1** students should see immediately that graph #1 is not a possible graph. There will probably be a discussion on graphs 2 and 3. Students should see that #3 has 0 workers working a finite amount of time to dig up the road and a finite amount of workers taking 0 time! The second graph would be the most realistic. The correct answer to question #2 on that same worksheet is (d). It is important that students see that the walk in the first hour was faster than the walk after they had rested. If students don't pick that answer, point out that the students walked 5 km in one hour going, they stopped for an hour, and then they walked back the same distance in 2 hours. They should be able to read speed from the graph.
GEE 21 Connection

This benchmark may be assessed with problems such as those shown below. They have been taken from the Illinois, Massachusetts, and Kentucky state tests.

1. Which of the graphs most accurately depicts the hourly wages earned with respect to the time worked?

A. 
B. 
C. 
D. 

2. The graphs below show the numbers of baskets made by Paul and Terrell during 5 basketball practice sessions. They each take 100 practice shots in each practice session. According to the information in the graphs, who was more successful at making baskets?

A. Paul did much better.
B. Terrell did much better.
C. Their scores appear to be about the same.*
D. More information is needed to make a decision.
3. During what 1-year period did automobile sales increase the most?

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<td>1970-80</td>
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<tr>
<td>1980-90</td>
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Illinois Standards Achievement Test

**Attributes of Student Work at the “Got-It” level**

Each of the three multiple-choice questions above require students to read information from a graph. In #1, students should recognize that if one is paid by the hour, then graph A is the only one that illustrates this scenario. In problem #2, there will be students who think Terrell has scored the most baskets because of the steepness of the linear segments or that Paul scored the most baskets because his line segments look higher. They need to look carefully at the vertical scaling and note that a “jag” has been made, and that Terrell started with 71 and ended with 80 much the same as Paul. This is a case where it is important that students carefully study all elements of the graph before answering. Question #3 can be answered by finding the amount for each year and then subtracting. It is more efficient to answer the question using the slopes of the line segments. A positive slope means an increase in sales, and the larger the slope the higher the sales.
GROWTH FUNCTIONS

(a)  

(b)  

(d)  

(c)  

(g)  

(o)
Below is a set of distance time graphs.
Which graphs can never represent the journey of a single vehicle. Why?

For the rest make up situations which could cause these graphs to be drawn.
This graph shows a car and motorbike, traveling along a country road.

Describe what is happening as fully as possible. Compare their distances apart and their speeds at different times.
1. When is the car ahead? The motorbike?
2. When are they furthest apart during the first 4 seconds?
3. Describe the speed of the car.
4. Describe the speed of the motorbike.
5. At what time does the motorbike catch up with the car?
6. How far ahead was the car in the beginning?
Lesson 8: Making Inferences

How many people does it take to dig up a two mile stretch of road in a given period of time? The graphs below can’t give you an exact answer. Which graph or graphs below can supply a reasonable estimate? Why did you choose the graph or graphs you did?

1. Time taken to dig up road
   Number of workers on job

2. Time taken to dig up road
   Number of workers on job

3. Time taken to dig up road
   Number of workers on job

2. The graph above describes the progress of a walk in the country. Which of the following describes the situation accurately?

   a. The hill is 5 km. high; it took 1 hour to get to the top and another hour to walk to the other side. It took 2 hours to get down the hill from the other side.

   b. The walk lasts 4 hours. In the first hour they walk for 5 km, building up speed as they go. For the next hour the pace is fast, and they walk another 5 km, but gradually over the next two hours the pace gets slower and slower until they stop.

   c. In 1 hour they walked 5 km; the second hour the pace was the same, but then it dropped to a rate of 2.5 kpm, and by hour 4 they weren’t walking anywhere at all.

   d. The walkers covered 5 km in the first hour, rested for an hour and then walked back half as fast.
GROWTH CURVES

Paul and Susan are two fairly typical people. The following graphs compare how their weights have changed during their first twenty years.

Answer the following questions about the graph above.
1. How much weight did each person put on during his/her middle and high school years (between the ages of 11 and 18)?
2. When did Paul weigh more than Susan? How can you tell?
3. When did they both weigh the same?
4. When was Susan putting on weight most rapidly? How can you tell from the graph?
5. When was Paul growing most rapidly?
6. Who was growing faster at the age of 14? How can you tell?
7. When was Paul growing faster than Susan? How can you tell?
GOING TO SCHOOL

Jane, Graham, Susan, Paul and Peter all travel to school along the same country road every morning. Peter goes in his dad’s car. Jane cycles and Susan walks. The other two children vary how they travel from day to day. The map above shows where each person lives.

The graph below describes each pupil’s journey to school last Monday.

a) Label each point on the graph with the name of the person its represents.

b) How did Paul and Graham travel to school on Monday? What was Jane’s speed?
c) Peter’s father is able to drive at 30 mph on the straight sections of the road, but he has to slow down for the corners. Sketch a graph on the axes below to show how the car’s speed varies along the route.
Answers to **Student Worksheet #1**

Students should see immediately that graph #1 is not a possible graph. There will probably be a discussion on graphs 2 and 3. Students should see that #3 has 0 workers working a finite amount of time to dig up the road and a finite amount of workers taking 0 time! The second graph would be the most realistic. The correct answer to question #2 on that same worksheet is (d). It is important that students see that the walk in the first hour was faster than the walk after they had rested.

Answers to **Student Worksheet 2**.

1. How much weight did each person put on during their middle and high school years (between the ages of 11 and 18)? Paul’s weight increased by 42 kilos during this period. Susan’s increased about 20.
2. When did Paul weigh more than Susan? How can you tell? Paul weighed more than Susan until about 9.7 years. They were about the same for the first year. From 9.5 to 15 years Susan weighed more than Paul. The graph of the person who weighs more is higher than the other.
3. When did they both weigh the same? They both weighed the same at 13 months, at about 9.7 years and again at 15 years.
4. When was Susan putting on weight most rapidly? Susan’s weight was increasing most rapidly at around 9.7 years. The graph is the steepest at that point.
5. When was Paul growing most rapidly? Paul’s weight was increasing most rapidly at around 15 years.
6. Who was growing faster at the age of 14? How can you tell? Paul. Susan had started to slow down. She had finished most of her growing.
7. When was Paul growing faster than Susan? How can you tell? From around 13 years on Paul was growing faster than Susan. His curve is increasing at an increasing rate while her rate is decreasing.

Answers to **Student Worksheet #3**.

a) Jane and Peter live six miles away from school. Susan and Graham both live 2 miles away. Paul lives 4 miles from school. Peter’s trip by car takes less than 20 minutes. Jane’s trip takes a little over 40 minutes. Therefore, the top left point is Peter, and the top right point is Jane. Paul lives 4 miles from school. His trip takes about 30 minutes. The middle point represents Paul. It takes Susan 40 minutes to walk to school. That particular day, Graham takes less than 20 minutes. The two bottom points represent Susan and Graham with Graham the left point and Susan the right one.
b) Paul and Graham probably rode their bicycles. Since Jane took 40 minutes to go 6 miles, she rode at about 9 miles per hour. (distance/time in hours). Susan’s walking speed is 3 miles per hour. Graham takes 20 minutes to go two miles. His speed is 6 miles per hour and Paul’s speed is 8 miles per hour.

c) 

![Peter's Journey to School graph](image-url)